ON THE EFFECTS OF GEOMETRIC NONLINEARITIES IN A FINITE-ELEMENT MODEL OF THE CAT EARDRUM

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ABSTRACT

Geometric nonlinearities are included in an existing finite-element model of the cat eardrum. The model is used to simulate the response of the eardrum to large static pressures after manubrial fixation. The nonlinear model agrees with the experimental observation that the middle ear is linear up to about 90 Pa. The response of the model becomes significantly nonlinear at higher pressures which has also been found in recent experimental work on the eardrum.

INTRODUCTION

Large static pressure differences across the eardrum are encountered in everyday life. They can, for example, be caused by the atmospheric pressure changes experienced in elevators and airplanes. Such pressure differences increase the hearing threshold by reducing the transmission of sound energy through the middle ear [1]. Since the eardrum is a primary determinant of the transmission characteristics of the middle ear, it is important both to measure and to model its mechanical behaviour.

The mechanical response of the eardrum to large static pressures has been studied experimentally using phase-shift shadow moiré topography [2,3], a non-contacting optical technique which allows the measurement of surface shapes. These studies reveal that the displacements of the eardrum are large compared to its thickness even in response to pressures as low as 100-200 Pa. In addition, they indicate that the response of the eardrum becomes stiffer at higher pressures, i.e., the displacements of the eardrum increase in smaller amounts as the pressure is increased in fixed amounts. The lack of proportionality between stimulus (pressure) and response (eardrum displacements) indicates that the behaviour of the eardrum is nonlinear. The types of nonlinearities involved (geometric, material or both) are not clearly understood.

Finite-element models of the eardrum would be useful in assessing the importance of various types of nonlinearities in the response of the eardrum. The objective of this work is to study the effects of geometric nonlinearities by including them in an existing linear finite-element model of the cat eardrum [4]. To simplify the situation, the initial modelling is focussed on the eardrum with manubrial fixation, thus eliminating the effects of the ossicular and cochlear loads.

DESCRIPTION OF THE MODEL

Model Geometry

The geometry of the eardrum model is shown in Fig. 1. Part (a) of that figure is a view of the eardrum in the plane of the annulus, and shows the division into triangular elements. Each triangle of the mesh represents a linear thin-shell element. The thick dark lines in Fig. 1(a) separate the pars tensa from the pars flaccida. Fig. 1(b) shows how the cone formed by the eardrum points medially. The overall 3-D shape of the eardrum is described by a normalized 'radius of curvature' equal to 1.19 [5].

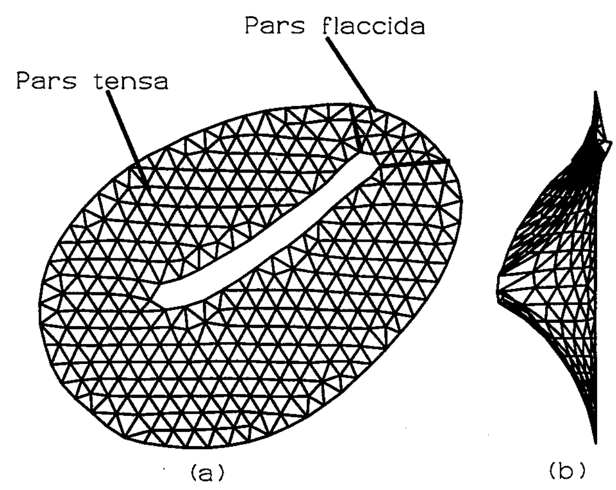

Fig. 1 Finite-element model of the cat eardrum with manubrial fixation. (a) Front view. (b) Side view.

Mechanical Properties

The material of the eardrum is modelled as being isotropic, homogeneous throughout its thickness and uniform across its surface. Since the purpose of this study is to assess the importance of geometric nonlinearities only, the material of the eardrum is assumed to be linearly elastic. This assumption is reasonable for small strains. The incremental Young's modulus of the portion of the model corresponding to the pars tensa is estimated from the data of Decraemer *et al.* [6] to be 8×10^5 Pa at the beginning of the stress-strain curve. The pars tensa has a thickness of 40 μm and a Poisson's ratio of 0.3. The pars flaccida has a thickness of 200 μm, an incremental Young's modulus of 1×10^5 Pa and a Poisson's ratio of 0.3. The manubrium is considered to be fully clamped, so the elements representing it are not included in the model. The periphery of the eardrum is also considered to be fully clamped.

Solution Procedure

Load-displacement curves were computed for a range of uniform static pressures from 0 to 1 kPa; the pressure was directed outward toward the ear canal. A combined incremental/iterative procedure was used for the simulations: the pressure was incremented in steps of 20 Pa and Newton-Raphson iterations were performed at each step. Iterations were stopped when the ratio of the Euclidean norm of the iterative displacement vector to that of the total displacement vector was less than 0.1%. The same criterion was also applied to the residual force vector to assure good force balance. Computations were done using the program I-DEAS from Structural Dynamics Research Corporation (SDRC).

RESULTS

Fig. 2 shows how the maximum displacement amplitude varies as a function of pressure after geometric nonlinearities are taken into account. For comparison, a linear solution was computed and is also shown in the figure.

DISCUSSION

Load-displacement characteristics of the eardrum model suggest that its mechanical behaviour is approximately linear for small pressure levels. For instance, at a pressure of 100 Pa, the linear solution is only 16% larger than the nonlinear solution. This compares well with the experimental finding [7] that the response of the middle ear is linear up to approximately 90 Pa, or 130 dB SPL. As the pressure is further increased in the nonlinear model, the displacements increase less than proportionally, so that at the maximum pressure studied here, 1 kPa, the nonlinear solution is 44% smaller than the linear one. This stiffening behaviour has also been found experimentally in cats in studies in which the manubrium was left mobile [3]. We will be extending the experimental measurements to include the fixed-manubrium situation. Such a study would be useful for refining our present model. In turn, a good model of the eardrum could aid in the interpretation of data obtained using clinical tympanometry where lesions of the eardrum are found to hinder detection of problems such as ossicular fixation which results in conductive hearing loss [8].

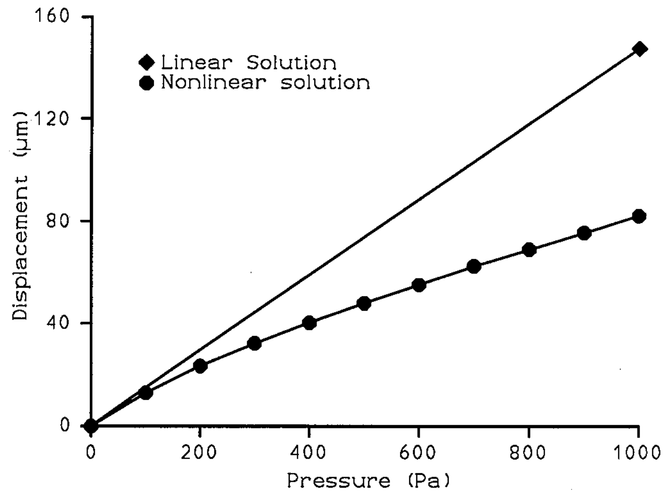

Fig. 2 Load-displacement curves for a point on the posterior region of the pars tensa.

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